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ISOSURFACE CONSTRUCTION FROM A DATA SET SAMPLED ON A FACE-CENTERED-CUBIC LATTICE

Abstract

A method for constructing the isosurface from a data set on a face-centered-cubic lattice has been developed. Generally, a three-dimensional scalar field is sampled on an orthogonal cubic lattice. And an algorithm called Marching Cubes is widely used for constructing the isosurface on this lattice. It is known, however, that data sampling of a three-dimensional scalar field on a face-centered-cubic lattice is theoretically optimum in terms of the density of the lattice points among the variety of sampling method. The isosurface on a face-centered-cubic lattice constructed by the proposed method is well approximated as a polyhedron by means of linear interpolation. Isosurfaces are constructed by the proposed method and by current ones in order to compare the properties of the surfaces. It was thus shown that the proposed method is more effective in terms of smoothness in appearance of isosurfaces.

keywords: *Isosurface construction, Face-centered-cubic lattice, Marching Cubes, Implicit surface*

1 INTRODUCTION

The technique of constructing and visualizing isosurface in three-dimensions (3D) is useful in many areas, e.g., *thresholded data* and *implicit surface*. In medical applications, visualizing the isosurface from a thresholded data set, such as CT images, is useful in understanding the anatomy [3, 25]. Implicit surface [8, 12, 13] is defined by a mathematical function, which can be applied to shape modeling of arbitrary geometries. And most of the methods generate such isosurface as connected polygons.

Marching Cubes [25] (MC) is one of the simple methods of them. Constructing the isosurface using MC is achieved by triangulating polygons in small cubic cell on a cubic lattice.

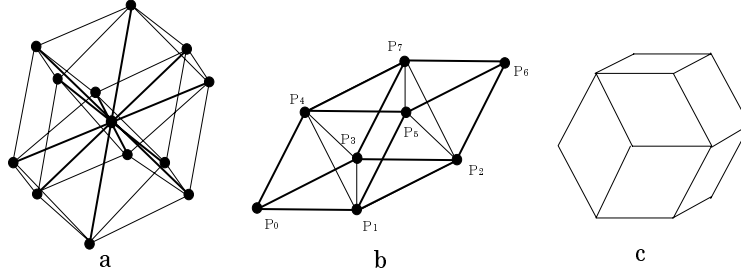


Fig. 1. FCC lattice: The 12 nearest neighboring points to a lattice point is shown by (a), the parallelepiped shown by (b) is utilized by the proposal method in order to generating cells, and both of (a) and (b) represent a FCC lattice, and (c) shows a rhombic dodecahedron, which is the Voronoi polyhedron of this lattice point

There are well known topological problems with MC although very simple and attractive algorithm. And many solutions have been reported to correct them [7, 16–19, 21, 26]. Methods of tetrahedral decomposition [1, 2] are one of the approaches to such solutions. They decompose a cubic cell of MC into several tetrahedral cells. Among them decomposing into five tetrahedrons [14] is very popular. Although the isosurfaces of these methods do not have the topological problems, the number of triangles tends to be larger.

The all applications mentioned above use the orthogonal cubic lattice for data sampling. This is likely to be associated with the current computer architecture, i.e., compatibility and portability of input and output devices, and memory layout. On the other hand, there are several works [4, 20], which take a different approach toward the orthogonal based methods. They employ a body-centered-cubic (BCC) lattice rather than an orthogonal one for a sampling lattice. There exist various types of lattice for data sampling. One of them, well-known face-centered-cubic (FCC) lattice (Fig. 1) is introduced in *sphere packing* problem [5] (Fig. 2). This problem demonstrates that data sampling on the lattice is theoretically optimum in terms of density of sampling points i.e., the number of spheres packed in the same volume. However, few reports have utilized a data set on this lattice for constructing the isosurface.

In light of the above background, the authors consider that the effectiveness of data sampling on a FCC lattice is related to the improvement in the resulting surfaces and have been develop a method for data sampling and constructing the isosurface on the FCC lattice [15]. The developed method decomposes a parallelepiped cell (Fig. 7) on a FCC lattice into two regular tetrahedral cells and an octahedral cell in order to construct the isosurface. Furthermore the authors examined effectiveness of the method by comparing the resulting surfaces of the method and two current methods, which are based on an orthogonal cubic lattice, including MC in terms of some geometrical properties.

2. RELATED WORKS

2.1 FCC

It is known that a FCC lattice is the natural expansion of a triangular lattice in two dimensional plane into three dimensional space. Figure 1(a) shows that each lattice point on FCC has the 12 nearest neighboring lattice points, from the same distance.

There are several works [5, 10, 22–24] on geometrical properties of various 3D lattices,

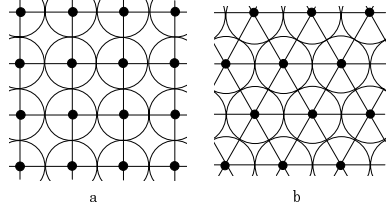


Fig. 2. The sphere packing problem: On the 2D plane, spheres are packed by using orthogonal lattice as (a), and by using triangular lattice as (b)

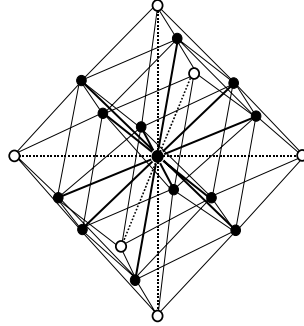


Fig. 3. Another type of neighboring points on FCC lattice: As to this type of them, 18 points are considered to neighbor to one lattice point

including a FCC lattice. One of them suggests that data sampling on a FCC lattice is optimum by calculating a *sampling distortion* on various types of lattice [5]. A distortion of each point $S = (x, y), x, y \in \mathbf{R}$ can be given as a square error d^2 (1) when S is in the *Voronoi* territory of a lattice point $Q = (\tilde{x}, \tilde{y}), \tilde{x}, \tilde{y} \in \mathbf{Z}$ under the condition that the areas (2D), the volumes (3D) of the Voronoi territories on every type of lattice are equal.

$$d^2 = (x - \tilde{x})^2 + (y - \tilde{y})^2 \quad (1)$$

Thus, the measurement of a sampling distortion on a lattice is given as a mean square error $\overline{d^2}$. In a case of 2D, data sampling on a triangular lattice makes $\overline{d^2}$ minimum and, therefore, is optimum. In a case of 3D, for the same reason, data sampling on a FCC lattice is optimum [5]. A rhombic dodecahedron, which is the shape of the Voronoi territory of a FCC lattice point, is shown in Fig.1(c).

The topological interpretation of a figure on a FCC lattice is investigated by the other group [22,23]. They describe that topological interpretation on a FCC lattice is simpler than that on an orthogonal cubic lattice by classifying configurations of points neighboring on one lattice point into some types in terms of digital geometry. There are two such types of neighboring points on a FCC lattice (Fig. 1(a) and Fig. 3) and four types of them on an orthogonal cubic lattice. Topological ambiguity increases with the number of these types and, then, complicates interpretation of topology on the lattice.

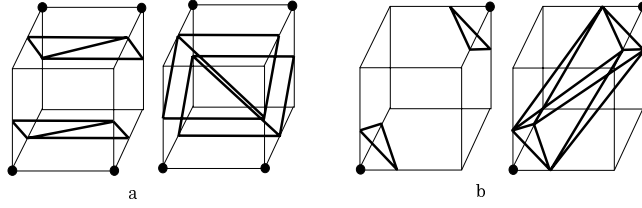


Fig. 4. Examples for ambiguous triangulation in a cubic cell: These let topology of resulting surface of MC not be consistent

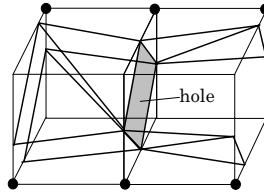


Fig. 5. The topological hole: Isosurface provided by MC may have this topological problem

2.2 MC methods

MC [25] is well-known for isosurface construction from a data set on a orthogonal cubic lattice. The resulting surface of MC is approximated as a polyhedron by generating triangles in cubic cells, whose vertices are eight adjacent lattice points. When one lattice point on the cell is internal of a figure and another adjacent lattice point is external, a vertex of the isosurface is generated along the edge between these two lattice points by way of linear interpolation. Polygons generated in the cell are triangulated in a certain way. There are 256 (i.e., 2^8) configurations of triangulation in a cubic cell. Accordingly, the previously defined table, which contains the correspondence between the configuration and the way of triangulation, can be used in order to reduce the computational cost for constructing the isosurface. Although defining a large table costs much, 256 types of configuration are integrated into 15 basic types by taking into account of symmetry, of rotation, and of the inversion.

Well-known problems are caused by ambiguity of triangulating the polygons in a cell. Fig.4 show their ambiguous situations. Topology of isosurface, which MC provides, is not consistent when isosurface contain their situations. Moreover, in a case where Fig. 4(a) is contained, these problem is more serious, because a topological hole may exist on the isosurface (Fig. 5). Many methods have been reported to correct these situations [7, 16, 17, 21]. And one of the simplest solutions uses a different triangulation in these cells for complementary symmetries of ambiguous face [21]. More complicated methods use sophisticated calculation to determine triangulation in these cells [7, 17].

On the other hand, methods of tetrahedral decomposition [1, 2] are one of the approaches to such solutions, therefore, can be considered a variation of MC. They decompose a cubic cell of MC into some tetrahedral cells. The resulting surfaces of these methods do not have the topological problems as MC has. And a tetrahedral cell only has 16 possible triangulations, which reduce to three by symmetry (Fig. 8). But the number of triangles of the surfaces tends to increase. Therefore the techniques of *mesh simplification* [6, 20], which reduce ver-

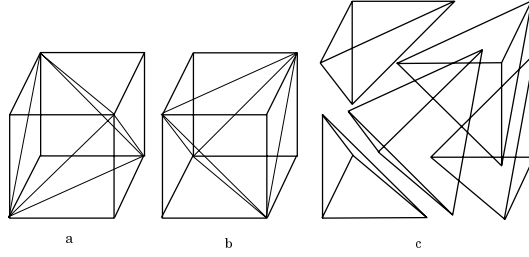


Fig. 6 . Decomposing cube into five tetrahedron

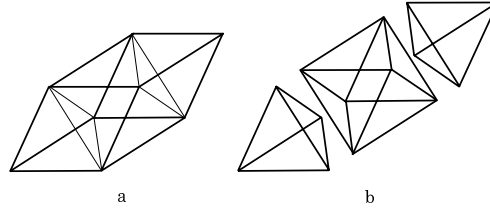


Fig. 7. Decomposing parallelepiped: The proposed method decomposes this into two regular tetrahedrons and a regular octahedron in order to construct the isosurface

tices and triangles, apply effectively in these cases. These methods are often called *Marching Tetrahedra* (MT). And one of them, which decompose into five tetrahedrons [14] as shown in Fig. 6(c), is popular, and so let us call this five tetrahedral version MT in the following description. MT uses two symmetrical decompositions of a cube and has to alternate between cubic cells, in order to align faces of the tetrahedral cells (Fig. 6(a),(b)). There is, therefore, another ambiguity that two different isosurface for a given data set can be generate.

3. ALGORITHM

The current paper focuses on the construction of implicit surface given by mathematical function based scalar field. Where, the data set can be sampled on an oblique cubic lattice, which Fig. 7(a) shows, rather than an conventional orthogonal cubic lattice. The detail of method for data sampling on the FCC lattice is discussed in the previous paper of the authors [15].

3.1. Isosurface construction

MT decomposes a cube, which is a cell of MC, into five tetrahedrons, whereas proposed method decomposes a parallelepiped in a FCC lattice into two regular tetrahedrons and a regular octahedron as shown in Fig. 7. There are three possible triangulations, which are identical with them of MT, in the tetrahedral cell (Fig. 8). And the octahedral cell has 64 possible triangulations, which can be reduced to seven, in nature, by symmetry (Fig. 9). There are, however, ambiguity in seven triangulations in the octahedral cell (Fig. 9(f),(g)), where topology of the isosurface of the proposed method may not be consistent. The proposed method does not generate, at least, the topological holes (Fig. 5), as MC does, by utilizing either triangulations (Fig. 9(f),(g)). Continuity of the isosurface is entirely retained, by using the proposed method. The one particular triangulation Fig. 9(f) of the two is employed here for this situation in the

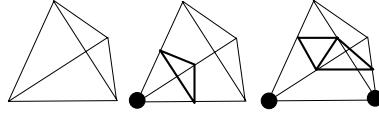


Fig. 8. Triangulation in tetrahedron: the three triangulations is reduced by symmetry, from 16 possible triangulations

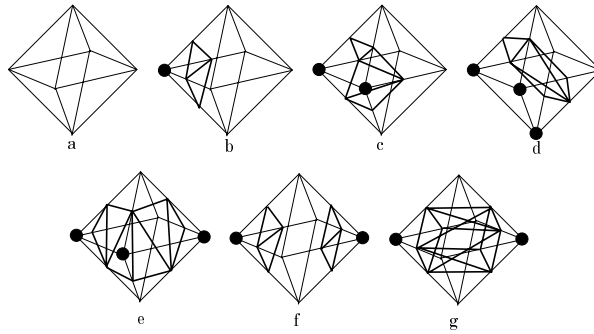


Fig. 9 . Triangulations in octahedron: These involve an ambiguity between (f) and (g)

current work, because Fig. 9(f) has less triangles than Fig.9(g) does. The similar method [17,21] can be applied to the problem of the ambiguity as the way applied to the problem in MC.

3.2. Implementation

The proposed method defines the tables, beforehand, which include the correspondence between configurations of vertices, and triangulations in a cell, for determining triangulations in order to reduce the computational cost.

Constructing the isosurface can be achieved by scanning the all sample points on a FCC lattice. The process for each sample point is explained below and (see Fig. 1(b)).

- Step 1) The position and value of the current sample point P_0 are calculated.
- Step 2) The positions and values of the other sample points P_1, \dots, P_7 , which locate on vertices of the parallelepiped for the current point P_0 , are calculated.
- Step 3) The parallelepiped is decomposed into two tetrahedral cells and an octahedral cell.
- Step 4) The positions of vertices of the isosurface are calculated along each edge, which crosses the isosurface, of these polyhedral cells.
- Step 5) The triangulations in the two tetrahedral cells and the octahedral cell are determined by referencing beforehand defined tables.

Until the all sample point are processed.

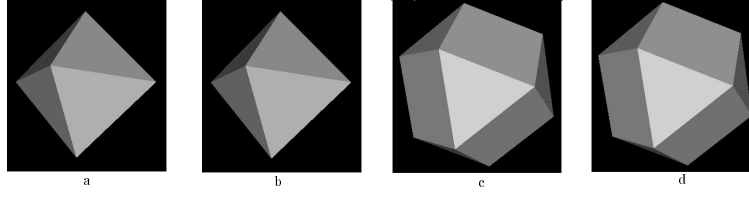


Fig. 10. Resulting surfaces in the first case: (a) is constructed by MC, (b) and (c) are done by MT, (d) is done by the proposed method, and they each have the only one lattice point in themselves

Table 1 . Comparison of the numbers of simplexes on Resulting surface in the first case

surface	method	vertices	edges	patches	internals
a	MC	6	12	8	1
b	MT	6	12	8	1
c		18	48	32	1
d	proposed	12	30	20	1

4. RESULTS AND DISCUSSION

4.1 Conditions of Comparison

As for an implicit surface, resulting surfaces by MC, MT and the proposed method are compared in the following. We employ the *metaball* functions [9] as an example mathematical form for implicit surface. When a scalar field is generated by several metaballs whose influence function is given by $f_i(x, y, z)$, a scalar value $F(x, y, z)$, $x, y, z \in \mathbf{R}$ is given by Equation (2) [9].

$$F(x, y, z) = \sum_i f_i(x, y, z) \quad (2)$$

The function $f_i(x, y, z)$ of a metaball which has a center (o_{xi}, o_{yi}, o_{zi}) and a strength d_i and a radius b_i is given by equation (3).

$$f_i(x, y, z) = \begin{cases} d_i(1 - 3(\frac{r_i}{b_i})^2) & (0 \leq r_i < \frac{b_i}{3}) \\ \frac{3d_i}{2}(1 - \frac{r_i}{b_i})^2 & (\frac{b_i}{3} \leq r_i < b_i) \\ 0 & (r_i \geq b_i) \end{cases} \quad (3)$$

$$r_i = \sqrt{(x - o_{xi})^2 + (y - o_{yi})^2 + (z - o_{zi})^2}$$

To compare, strictly, the resulting surfaces of the proposed method and of current methods, the densities of sampling points on an FCC and on an orthogonal cubic lattice are made even. That is, the distance between neighboring sample points is varied so that, for both types of lattice, the Voronoi polyhedrons of lattice points have the same volume [5, 10]. These polyhedrons are a rhombic dodecahedron (Fig. 1(c)) in the case of the FCC lattice and a cube in the case of the orthogonal cubic lattice.

4.2 Resulting surfaces

The resulting surfaces of each method in the first case are shown in Fig. 10. In the first case, the scalar field is generated by a metaball whose radius is small enough. These surfaces are

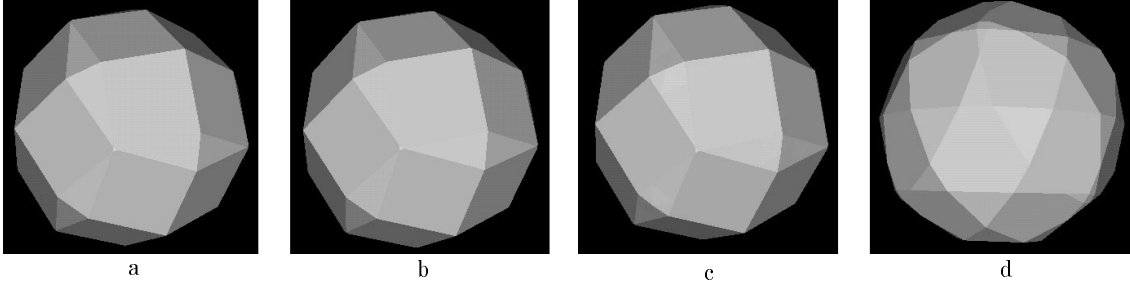


Fig. 11. Resulting surfaces in second case: (a) is constructed by MC, (b) and (c) are done by MT, (d) is done by the proposed method, and it seems that (d) is the most smooth among the three surfaces

Table 2. Comparison of the numbers of simplexes on Resulting surface in the second case

surface	method	vertices	edges	patches	internals
a	MC	54	156	104	19
b	MT	102	300	200	19
c		138	408	272	19
d	proposed	108	318	212	19

the simplest for each method because there is the only one sample point inside of each surface. Fig. 10(a) shows the resulting surface of MC. And (b) and (c) show surfaces of MT. And (d) shows a surface of our method. The number of simplexes and internal points in the surfaces (a), (b), (c) and (d) are described in Table 1. In the case of MT, because of two different ways of decomposing a cube, there are two shapes of surface, (b) and (c).

In the second case, a scalar field is generated by a metaball whose radius is larger than the radius in the first case. The resulting surfaces of each method are shown in Fig. 11. And the number of simplexes and internal points in the surfaces (a), (b), (c), and (d) are described in Table 2. The surfaces (a) of MC and (b) and (c) of MT are almost the same in appearance, however, the number of simplexes of them are all-different. In terms of smoothness in appearance, the surface (d) of our method is considered the most smooth than the others in this case.

Finally, Figures 12 show part of the each resulting surface from a scalar field generated by 15 metaballs. Each metaball has a center, strength and a radius at random. Since there is no remarkable difference between the two surfaces of MT, in this case, the only one surface is shown for MT. The number of simplexes and internal points in surfaces constructed by each method are listed in Table 3, additionally, the processing times for construction of each method are also listed in Table 3. In this case also, the isosurface of our method can be strictly compared with the surfaces of the other methods in terms of the density of sample points, because the numbers of the internal points of each method are almost even. The resulting surface of our method seems to be smoother than the other surfaces in appearance. And the number of simplexes of our method and that of MT are almost even.

5. CONCLUSION AND FUTURE WORKS

Proposed method constructs the isosurfaces on a FCC lattice by using two different shapes of polyhedral cell, i.e., the regular tetrahedral cell and the regular octahedral cell. The resulting

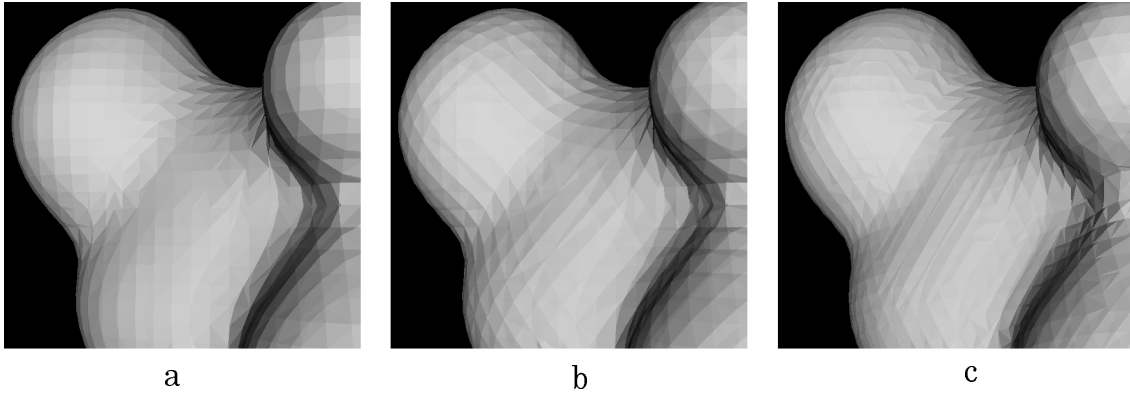


Fig. 12. Resulting surfaces in third case: To compare the smoothness in appearance clearly, parts of the surface are shown, (a) is constructed by MC, (b) is done by MT, (c) is done by the proposed method

Table 3 . Comparison the numbers of simplexes on resulting surface in the third case

surface	method	vertices	edges	patches	internals	time
a	MC	3280	9828	6552	6037	3448
b	MT	7900	23688	15792	6037	6555
c	proposed	7352	22044	14696	5997	4027

surface does not have the topological holes, which MC does, although there is a topological ambiguity in the triangulation in the octahedral cell. As for comparison of properties of the resulting surfaces, the effectiveness of the method can be particularly confirmed in Fig. 11,12 in terms of the smoothness in appearance. The triangles generated by our method is as much as that of MT, but the methods of mesh simplification [6, 20] can also be applied to the resulting surface of our method in order to reduce them. As for data sampling on a FCC lattice, there is no disadvantage for constructing the implicit surface. Computational costs are fairly compatible in all three method as described in Tab. 3.

Future works include investigation the effectiveness of our method by using other criteria, e.g., the *aspect ratio* of triangles [6, 20], taking into account various cases of resulting surface.

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